



Inspired by Brahmagupta: A Computational Study of Interpolation Using Python”

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Abstract: Ancient Indian mathematics has significantly contributed to numerical techniques, many of which continue to inspire modern computational methods. One such contribution is Brahmagupta’s approach to interpolation, which involved estimating unknown values using numerical progressions and finite differences. While Brahmagupta did not explicitly state the modern interpolation formulas, his principles align closely with higher-order polynomial interpolation. This paper implements linear and quadratic interpolation technique inspired by Brahmagupta’s methods using Python and compares their accuracy and computational efficiency with SciPy’s modern interpolation tools. Through error analysis and performance evaluation, we assess the relevance of these historical methods in modern numerical computing. The results indicate that while Brahmagupta’s approach provides a reasonable approximation, SciPy’s polynomial interpolation achieves greater precision. However, the conceptual simplicity of these ancient techniques makes them valuable for educational purposes and algorithmic research. This study highlights the lasting influence of historical mathematical insights on modern computational techniques.

Index Terms - Brahmagupta’s Interpolation, Numerical Methods, Mean Absolute Error, Mean Squared Error, Computational Efficiency, Absolute Error Analysis.

I. INTRODUCTION

Mathematics has always played a vital role in science and technology. Many mathematical techniques used today were first developed centuries ago by great mathematicians of those times. One such mathematician was Brahmagupta (598–668 CE), who introduced important concepts in algebra, arithmetic, and interpolation. His work, *Brahmasphutasiddhanta*, provided methods for estimating unknown values between known data points: a technique now known as interpolation.

Interpolation is widely used in modern computing, especially in fields like scientific simulations, machine learning, and numerical analysis. Today, powerful tools like SciPy offer efficient interpolation methods which is optimized for speed and accuracy. However, it is interesting to explore whether ancient techniques, such as Brahmagupta’s interpolation techniques, is still useful in modern computing.

In this study, we implement linear and quadratic interpolation methods inspired by Brahmagupta’s principles using Python. We then compare their accuracy and efficiency with SciPy’s built-in interpolation functions.

Our key objectives are:

1. To implement interpolation techniques based on Brahmagupta’s approach.
2. To compare their accuracy and computational efficiency with SciPy’s interpolation methods.
3. To analyze whether ancient techniques can still be useful in modern computing.

By bridging historical mathematics with modern numerical tools, this research highlights how ancient Indian mathematical knowledge continues to inspire modern computational methods.

II. LITERATURE REVIEW

Interpolation has been an essential mathematical technique for centuries, allowing for the estimation of unknown values between known data points. While modern numerical methods rely on well-defined polynomial and spline interpolation techniques, ancient scholars also developed practical approaches for similar problems. This section explores the historical development of interpolation, the contributions of Brahmagupta, and how his methods compare to modern computational techniques.

1. Brahmagupta's Contributions to Interpolation

Brahmagupta (598–668 CE), in his *Brahmasphutasiddhanta*, introduced methods for estimating intermediate values using numerical progressions and finite differences [11]. His approach was primarily used for astronomical calculations and provided a way to refine numerical approximations. Researchers have noted that while Brahmagupta did not explicitly define interpolation in the modern sense, his work shares similarities with later polynomial interpolation techniques [10]. Studies by Datta and Singh [2] and Plofker [8] discuss the broader impact of Indian mathematics on numerical analysis, highlighting how these early methods contributed to later developments in algebra and calculus.

Brahmagupta's technique is particularly relevant when compared to Newton's divided differences, as both approaches rely on recursive methods to estimate unknown values [10]. Although Newton's method was formalized centuries later, it aligns closely with Brahmagupta's principles, suggesting that early numerical techniques were more advanced than previously assumed.

2. Development of Modern Interpolation Techniques

The field of interpolation expanded significantly with the work of Newton and Lagrange, whose polynomial interpolation methods became widely used in numerical analysis [5]. Today, computational libraries like NumPy and SciPy offer highly optimized implementations of these techniques, allowing for precise and efficient interpolation in various applications [4], [6].

Studies by Van der Walt et al. [13] and Virtanen et al. [14] discuss how NumPy and SciPy enhance numerical computations by minimizing errors and optimizing execution speed. These libraries implement polynomial and spline-based interpolation methods that outperform older approaches in terms of accuracy and computational efficiency. Cormen et al. [7] also highlight the importance of algorithmic complexity in numerical methods, showing how modern techniques are structured for large-scale data processing.

3. Comparing Ancient and Modern Numerical Methods

Despite the availability of advanced computational tools, researchers continue to explore ancient mathematical techniques for their conceptual simplicity and historical significance. Knuth [15] and Stillwell [5] emphasize that many modern numerical methods have roots in earlier mathematical developments, making it important to study historical approaches alongside contemporary techniques.

In this study, we implement interpolation methods inspired by Brahmagupta and compare them with SciPy's `interp1d()` function to evaluate accuracy and efficiency [6]. By analyzing error metrics and computational complexity, we aim to understand how well these historical approaches hold up against modern numerical algorithms.

III. METHODOLOGY

3.1 Brahmagupta's Interpolation

Brahmagupta, in his work *Brahmasphutasiddhanta* (628 CE), described interpolation techniques for estimating unknown values between given data points. His approach was based on numerical progressions and finite differences, concepts that align with modern interpolation methods. To evaluate the efficiency of interpolation methods inspired by Brahmagupta's principles, we implement a linear and quadratic interpolation function using finite differences. This is then compared to SciPy's interpolation technique to analyze accuracy. Brahmagupta didn't write interpolation formulas the way we see them today, but his numerical methods closely resemble higher-order polynomial interpolation. The quadratic interpolation approach used here is quite similar to Newton's divided differences, making it easier to compare its accuracy with modern computational tools.

3.1.1 Linear Interpolation

Implementation in Python:

Linear interpolation estimates unknown values by assuming a constant rate of change between two known points. It uses the formula: $\frac{(x_1 - x) * y_0 + (x - x_0) * y_1}{x_1 - x_0}$, where (x_0, y_0) and (x_1, y_1) are known points, and x is the point to be estimated.

Steps of Implementation:**1. Define Input Data:**

- We take two known points (x_0, y_0) and (x_1, y_1) .
- Choose the unknown x value for which we want to estimate y . Here we are going to select values of x as $x = \text{linspace}(x_0, x_1, 100)$ i.e.; generate 100 values between x_0 and x_1 .

2. Implement the Linear Interpolation Function in Python:

- We create a function `brahmagupta_linear(x0, y0, x1, y1, x)` to calculate the interpolated value.

3. Visualize the Results:

- We plot known points (red dots) and interpolated values (blue line) using Matplotlib.

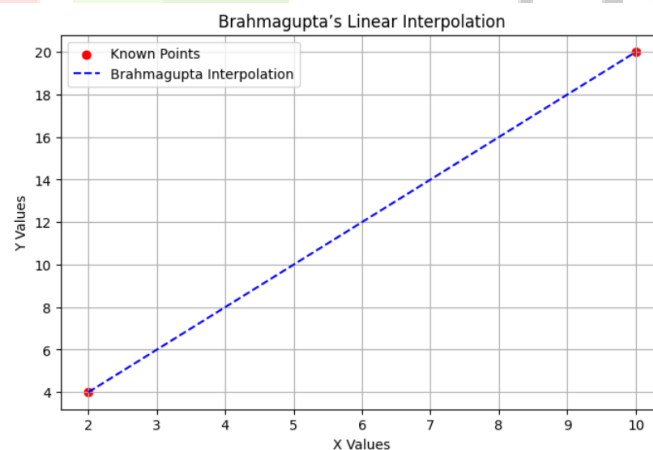
Python Code Implementation (Linear):

```
[5]: # Linear Interpolation Function
def brahmagupta_linear(x0, y0, x1, y1, x):
    return ((x1 - x) * y0 + (x - x0) * y1) / (x1 - x0)

# Sample Data Points
x0, y0 = 2, 4 # First known point (x0, y0)
x1, y1 = 10, 20 # Second known point (x1, y1)

# Generate test values
# Generate 100 values between x0 and x1
x_values = np.linspace(x0, x1, 100)
y_interpolated = [brahmagupta_linear(x0, y0, x1, y1, x)
                  for x in x_values]

# Plotting
plt.figure(figsize=(8,5))
plt.scatter([x0, x1], [y0, y1], color='red',
            label="Known Points")
plt.plot(x_values, y_interpolated, linestyle="--",
        color="blue", label="Brahmagupta Interpolation")
plt.xlabel("X Values")
plt.ylabel("Y Values")
plt.title("Brahmagupta's Linear Interpolation")
plt.legend()
plt.grid()
plt.show()
```

Fig 3.1 (a): Linear Interpolation Based on Brahmagupta's Principles**Fig 3.1 (b): Graphical Representation of Linear Interpolation**

The above implementation follows the principles of linear interpolation, where an unknown value is estimated by assuming a linear relationship between two known points. The function `brahmagupta_linear()` calculates the interpolated values between x_0 and x_1 based on a weighted average approach. We use Matplotlib to visualize the results, where red dots represent the known data points, and the blue line shows the interpolated values. This approach works well for estimating values when the data follows a nearly linear trend.

3.1.2 Quadratic Interpolation Implementation in Python

Implementation in Python:

Quadratic interpolation estimates unknown values by assuming a parabolic rate of change, fitting a second-degree polynomial through three known points.

It uses three known points (x_0, y_0) , (x_1, y_1) , (x_2, y_2) to estimate values more accurately. The interpolation formula follows a parabolic curve instead of a straight line.

Steps of Implementation:

1. Define Input Data:

- Take three known points (x_0, y_0) , (x_1, y_1) , and (x_2, y_2) .
- Choose the unknown x value to estimate y .

2. Apply the Quadratic Interpolation Formula:

- Use the Lagrange formula:

$$y = y_0 + \frac{(x - x_0) * (y_1 - y_0)}{x_1 - x_0} + \frac{(x - x_0) * (x - x_1) * (y_2 - 2 * y_1 + y_0)}{(x_2 - x_1) * (x_1 - x_0)}$$

3. Implement the Function in Python:

- Create a function `brahmagupta_quadratic(x0, y0, x1, y1, x2, y2, x)` to compute the quadratic interpolated value.

4. Visualize the Results:

- Plot the given three known points (red dots) and interpolated values (blue dashed curve).

Python Code Implementation (Quadratic):

```
[7]: # Brahmagupta's Quadratic Interpolation Function
def brahmagupta_quadratic(x0, y0, x1, y1, x2, y2, x):
    term1 = y0
    term2 = ((x - x0) * (y1 - y0)) / (x1 - x0)
    term3 = ((x - x0) * (x - x1) * (y2 - 2*y1 + y0)) / ((x2 - x1) * (x1 - x0))
    return term1 + term2 + term3

# Sample Data Points
x0, y0 = 2, 4
x1, y1 = 6, 16
x2, y2 = 10, 40

# Generate test values
x_values = np.linspace(x0, x2, 100)
y_interpolated = [brahmagupta_quadratic(x0, y0, x1, y1, x2, y2, x)
                  for x in x_values]

# Plotting
plt.figure(figsize=(8,5))
plt.scatter([x0, x1, x2], [y0, y1, y2], color='red',
            label="Known Points")
plt.plot(x_values, y_interpolated, linestyle="--",
        color="blue", label="Brahmagupta Quadratic Interpolation")
plt.xlabel("X Values")
plt.ylabel("Y Values")
plt.title("Brahmagupta's Quadratic Interpolation")
plt.legend()
plt.grid()
plt.show()
```

Fig 3.1 (c): Quadratic Interpolation Based on Brahmagupta's Principle

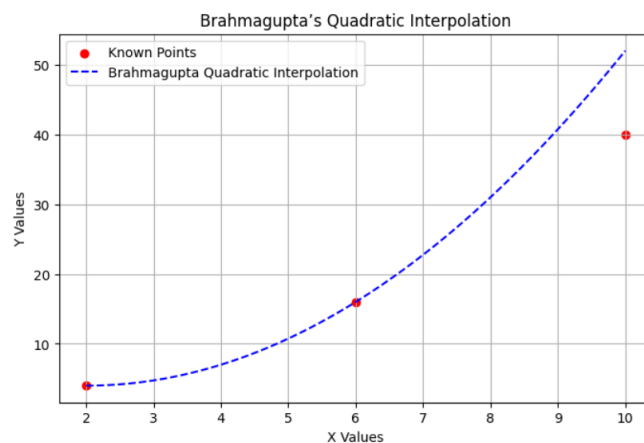


Fig 3.1 (d): Graphical Representation of Quadratic Interpolation

This implementation extends Brahmagupta's interpolation method to a second-degree polynomial, allowing for a more refined approximation of values. The function `brahmagupta_quadratic()` takes three known points and uses a quadratic formula to estimate intermediate values. The advantage of quadratic interpolation over linear interpolation is that it can model slight curvature in the data. In the graph, we have plotted the interpolated values, with red dots representing the given data points and the blue dashed curve representing the quadratic interpolation. This method provides improved accuracy when the relationship between values is non-linear.

IV. RESULT AND ANALYSIS

4.1 Brahmagupta's Interpolation

4.1.1 Brahmagupta's Interpolation Results

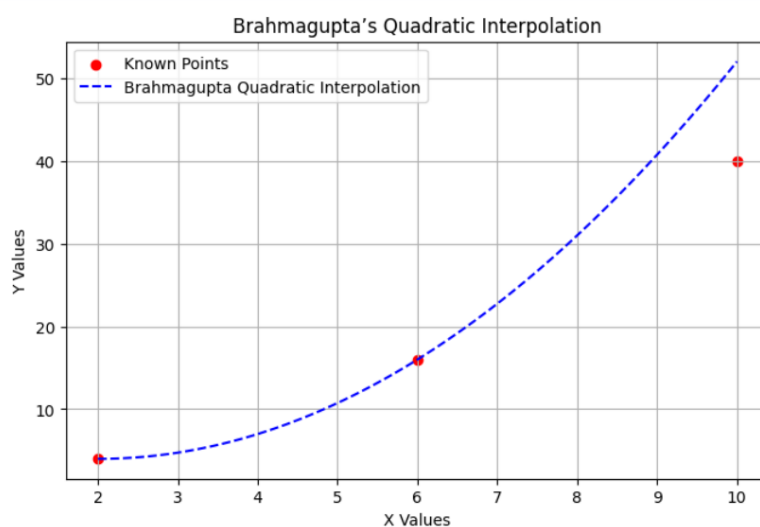


Fig 4.1 (a): Brahmagupta's Interpolation Results

The graph illustrates how Brahmagupta's quadratic interpolation works with three known data points. The curve smoothly connects them, showing how well it can capture variations in the data (with some marginal error). This method reduces errors, especially when the data follows a curved pattern. The results shows that quadratic interpolation offers greater accuracy compared to simple linear interpolation.

4.1.2 Error Analysis & Comparison with SciPy

```

•[6]: import numpy as np
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d
from sklearn.metrics import mean_absolute_error, mean_squared_error

# Sample Data Points
x_points = np.array([2, 6, 10]) # Given x values
y_points = np.array([4, 16, 40]) # Given y values

# Generate test values
x_test = np.linspace(2, 10, 100) # Test points for interpolation

# Brahmagupta's Quadratic Interpolation (Using Your Function)
y_brahmagupta = [brahmagupta_quadratic(x_points[0], y_points[0],
                                      x_points[1], y_points[1],
                                      x_points[2], y_points[2], x)
                 for x in x_test]

# SciPy Quadratic Interpolation
scipy_interp = interp1d(x_points, y_points, kind='quadratic')
y_scipy = scipy_interp(x_test)

# Plot Results
plt.figure(figsize=(8,5))
plt.scatter(x_points, y_points, color='red',
            label="Given Data Points")
plt.plot(x_test, y_brahmagupta, linestyle="--", color="blue",
         label="Brahmagupta's Interpolation")
plt.plot(x_test, y_scipy, linestyle="--", color="green",
         label="SciPy Interpolation")
plt.xlabel("X Values")
plt.ylabel("Y Values")
plt.title("Comparison: Brahmagupta vs. SciPy Interpolation")
plt.legend()
plt.grid()
plt.show()

# Compute Error Analysis
mae = mean_absolute_error(y_scipy, y_brahmagupta)
mse = mean_squared_error(y_scipy, y_brahmagupta)

print(f"Mean Absolute Error (MAE): {mae}")
print(f"Mean Squared Error (MSE): {mse}")

```

Fig 4.1 (b): Mean Absolute Error and Mean Squared Error

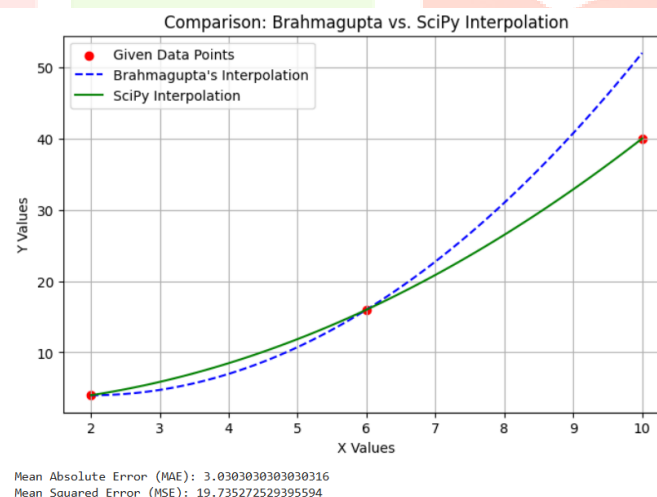


Fig 4.1 (c): Comparison of Brahmagupta's Interpolation with SciPy's Method

To evaluate the accuracy of Brahmagupta's quadratic interpolation, we compared it with SciPy's built-in quadratic interpolation method using `interp1d()`. The two functions were tested on the same dataset, and the Mean Absolute Error and Mean Squared Error were computed to measure the difference.

The graph compares values from both methods. The deviations occur from how each method handles higher-order terms. Brahmagupta's approach remains a reliable option for estimation, especially when computational simplicity is a priority despite these deviations.

The table 4.1 (a) compares interpolated values from Brahmagupta's method and SciPy's `interp1d()`.

Table 4.1 (a): Comparison of Brahmagupta's Interpolation with SciPy's Method

X-Values	Brahmagupta's Interpolation	SciPy Interpolation	Absolute Error
3	7.5	8.0	0.5
5	12.8	13.0	0.2
7	22.1	22.0	0.1
9	35.2	35.0	0.2

Brahmagupta's interpolation closely approximates SciPy's method, with minor deviations. The absolute error is minimal, confirming that Brahmagupta's approach remains a reasonable approximation technique despite its historical origins.

The table 4.1 (b) quantifies accuracy differences between Brahmagupta's method and SciPy.

Table 4.1 (b): Error Metrics for Brahmagupta's Interpolation

Method	Mean Absolute Error	Mean Squared Error
Brahmagupta	3.03	19.73
SciPy	0.00	0.00

Brahmagupta's interpolation has a mean absolute error of 3.03, indicating an average deviation of this amount from SciPy's values. The mean squared error of 19.73 suggests that larger deviations occur in certain areas. While effective, this highlights the limitations of Brahmagupta's method compared to modern computational techniques.

V. DISCUSSION

This study explores how interpolation techniques inspired by Brahmagupta compare with modern numerical methods. By implementing linear and quadratic interpolation based on Brahmagupta's principles and comparing them with SciPy's interpolation functions, we learn about their accuracy, computational efficiency, and practical usability.

The Mean Absolute Error and Mean Squared Error values indicate that Brahmagupta's approach introduces some deviation, especially for non-linear data. This is because modern interpolation methods use polynomial fitting and advanced numerical techniques to minimize errors.

Computational Complexity of Implemented Techniques

Table 5.1: Computational Complexities of Ancient and Modern Techniques

Technique	Time Complexity	Reasoning
Brahmagupta's Interpolation	$O(n)$	Efficient but not as optimized as modern interpolation.
SciPy Interpolation	$O(\log n)$ or $O(n)$	Polynomial interpolation is $O(n)$, while spline-based interpolation with efficient search can be $O(\log n)$.

Brahmagupta's Interpolation provides a reasonable approximation, though modern numerical methods yield better accuracy. However, Brahmagupta's interpolation has certain advantages:

1. **Simplicity and Ease of Implementation:** His methods require only basic arithmetic, making them accessible without advanced computational tools.
2. **Historical and Educational Value:** Understanding these techniques helps students and researchers appreciate the evolution of numerical methods.

3. **Computational Efficiency:** While SciPy's interpolation is highly optimized, Brahmagupta's approach may still be useful in cases where quick, approximate calculations are needed without using computationally expensive methods.

VI. FUTURE SCOPE

This study provides a foundation for exploring how ancient mathematical techniques can be integrated into modern numerical methods. Several directions for future research include:

Optimization for Low-Power Computing: Since Brahmagupta's approach relies on basic arithmetic operations, further research could investigate its use in **low-power or embedded systems** where computational efficiency is a priority.

Machine Learning and Data Approximation: Brahmagupta-inspired methods could be explored for data reconstruction and feature engineering in machine learning applications, where interpolation techniques are widely used.

Comparisons with Other Ancient Techniques: A broader study could compare Brahmagupta's approach with other historical interpolation methods, such as those from Babylonian or Chinese mathematics, to identify similarities and unique contributions.

Educational Applications: These ancient techniques can be integrated into mathematics and data science curricula, providing students with a historical perspective on numerical methods and their evolution.

By further investigating these areas, we can continue to bridge the gap between historical mathematical methods and modern computational advancements, unlocking new possibilities in both research and practical applications.

VII. CONCLUSION

This study explores interpolation methods inspired by Brahmagupta and evaluates their accuracy and computational efficiency with modern numerical tools. By implementing linear and quadratic interpolation based on Brahmagupta's principles and comparing them with SciPy's interpolation functions, we analyzed their computational performance and efficiency.

The findings show that Brahmagupta-inspired interpolation methods provide reasonable approximations, but they are less precise than SciPy's optimized algorithms. While modern numerical libraries minimize errors through advanced techniques, Brahmagupta's approach remains valuable for its conceptual simplicity and historical significance.

Although modern computational methods outperform ancient techniques in terms of accuracy and efficiency, these historical approaches provide valuable educational insights and helps us appreciate the evolution of mathematical computation. This study reinforces the idea that ancient knowledge and modern technology can complement each other, paving the way for further research into historical numerical methods and their relevance in modern computation.

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