



Impacts Of Temperature Gradients And Diffusion-Thermo Effects On Rayleigh Benard Double Diffusive Convection In Two Layer System With Darcy Brinkmann Model : Rigid Free Boundaries Withrout Surface Tension

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Abstract

The influence of the Non-Darcian effect and Diffusion – Thermo (Dufour effect) effect on Two-Component Rayleigh Benard Marangoni (TCRBM) convection in a composite layer, has been investigated analytically. For the fluid layer, the upper boundary is free with surface tension depending on both concentration and temperature, for the porous layer, the lower boundary is rigid and both boundaries are insulating to both heat and mass. At the interface, the normal velocity, normal stress, shear stress, mass, mass flux, heat, heat flux are assumed to be continuous. The Darcy-Brinkmann model term and the Dufour term are employed for the momentum and energy equation respectively. The impact of various physical parameters on the onset of TCRBM convection is investigated in detail and results are presented graphically.

Keywords: Rayleigh Benard Marangoni Convection, Dufour effect, Two – Component Convection, Composite layer.

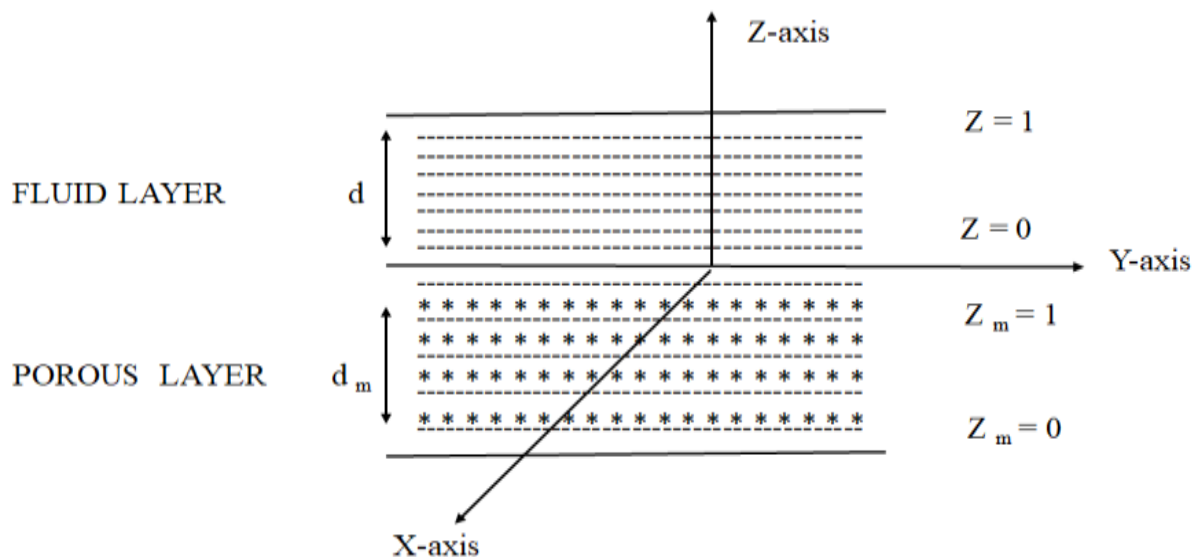
INTRODUCTION

Rayleigh Benard Marangoni Two-Component convection has wide applications in industries such as solidification, crystal growth, waste disposals, solar ponds, energy reservoirs and etc. In a Composite layered system, the exchange of light fluid particles to its top and heavy fluid particles to its bottom this process will happen continuously because of the presence of gravitational forces. Y.H. Gangadharaiah (2015) has considered composite layer to investigate Benard Marangoni Convection (BMC) in detail in a porous medium, which is anisotropic. The upper boundary of the composite layer is atmospheric-free and assumed to be deformable by making use of the Regular Perturbation Method, he solved the eigenvalue problem with perturbed parameter as " α " and also conclude that parameters of both thermal anisotropic and mechanical anisotropic enhanced by the influence of stabilizing factors and destabilizing factors. Alexander Alexeev et.al (2005) has considered heated walls in thin liquid films to investigate the most important

concepts of heat transfer and Marangoni Convection along with topography has been done both numerically and experimentally. For fluid that is incompressible, they are developed an algorithm relates to the finite difference method to investigate Navierstokes and energy equations. This method is used to investigate the film rupture and flow patterns affected by the forces of thermo capillary finally, they were found that in liquids enhancement of heat transfer occurs because of thermo capillary convection so this type of effect purely depends on the wall surface shape. M.C.Tsai and Sindo Kou (1984) have done very good work on Marangoni Convection with the help of weld pools which is surface free, they were developed a two-dimensional model for steady state to investigate fluid flow and heat transfer of heat with the effects of Marangoni in the weld pools. Sumithra.R et.al (2021) have considered a Composite layered system to investigate the impacts of both the Thermo diffusion effect (Soret effect) and Non-Darcian effects on Rayleigh Benard Convection, Both the boundaries of the Composite layered system are assumed to be rigid. The thermal critical Rayleigh number is found by approaching the regular perturbation technique which was the main criterion for convection finally they concluded that a composite layered system is stabilizes in the presence of a thermal diffusion effect. Malashetty et.al (2011) have been investigated the impacts of cross-diffusion effects means the combined effect of the Dufour effect and Soret effect on two-component (double diffusion) convection, for this binary Maxwell fluid is considerable and also they were investigated analytically by making use of stability analysis (linear and nonlinear) generally the technique of normal mode and truncated representation of Fourier series is used to analyze the stability of linear and nonlinear respectively. The impacts of the normalized porosity parameter, relaxation parameter, lewis number, vadasz number, Dufour parameter, Solutal Rayleigh number and Soret parameter on both stationary and oscillatory convections are depicted graphically.

MATHEMATICAL FORMULATION

The physical model under the consideration of composite layer system with a horizontal two – component fluid saturated, incompressible, isotropic and sparsely packed porous layer is of thickness d_m and a fluid layer of thickness d . Both the boundaries of the composite layer system are considered to be rigid and these boundaries are acts like heat and mass insulators along with maintained, distinct concentration and temperature.



The origin point of the Cartesian coordinate system is taken exactly at the intersection of porous and fluid layer along with direction of z – axis is vertically upwards. In addition to that, for the effect of density variation, Boussinesq approximation is included. Under these following assumptions, the governing equations are, the continuity, momentum, temperature, concentration and state equations as follows.

For Region – 1(fluid layer)

$$\nabla_{FL} \cdot \vec{V}_{FL} = 0$$

$$\rho_{FLO} \left[\frac{\partial \vec{V}_{FL}}{\partial t_{FL}} + (\vec{V}_{FL} \cdot \nabla_{FL}) \vec{V}_{FL} \right] = -\nabla_{FL} P_{FL} + \mu_{FL} \nabla_{FL}^2 \vec{V}_{FL} - \rho_{FL} g_{FL} \hat{k}$$

$$\frac{\partial T_{FL}}{\partial t_{FL}} + (\vec{V}_{FL} \cdot \nabla_{FL}) T_{FL} = k_{FL} \nabla_{FL}^2 T_{FL} + k_{FLT} \nabla_{FL}^2 C_{FL}$$

$$\frac{\partial C_{FL}}{\partial t_{FL}} + (\vec{V}_{FL} \cdot \nabla_{FL}) C_{FL} = k_{FLS} \nabla_{FL}^2 C_{FL}$$

$$\rho_{FL} = \rho_{FLO} [1 + \alpha_{FLS} (C_{FL} - C_{FLO}) - \alpha_{FLt} (T_{FL} - T_{FLO})]$$

For Region – 2(porous layer)

$$\nabla_{Fm} \cdot \vec{V}_{Fm} = 0$$

$$\left[\frac{\rho_{Fm0}}{\phi_{Fm}} \right] \frac{\partial \vec{V}_{Fm}}{\partial t_{Fm}} = -\nabla_{Fm} p_{Fm} - \frac{\mu_{Fm}}{K} \vec{V}_{Fm} + \mu_{Fm} \nabla_{Fm}^2 \vec{V}_{Fm} - \rho_{Fm} g_{Fm} K_{\widehat{Fm}}$$

$$A \left[\frac{\partial T_{Fm}}{\partial t_{Fm}} \right] + (\vec{V}_{Fm} \cdot \nabla_{Fm}) T_{Fm} = \kappa_{Fm} \nabla_{Fm} T_{Fm} + \kappa_{FmT} \nabla_{Fm}^2 C_{Fm}$$

$$\phi_{Fm} \left[\frac{\partial C_{Fm}}{\partial t_{Fm}} \right] + (\vec{V}_{Fm} \cdot \nabla_{Fm}) C_{Fm} = \kappa_{Fm} \nabla_{Fm}^2 C_{Fm}$$

$$\rho_{Fm} = \rho_{Fm0} [1 - \alpha_{Ftm} (T_{Fm} - T_{Fm0}) + \alpha_{Fsm} (C_{Fm} - C_{Fm0})]$$

Where, $\vec{V} = (u, v, w)$ is fluid layer velocity vector, t is the time, ρ_{FLO} is referring fluid density, μ_{FL} is the viscosity of fluid, P_{FL} is pressure, ρ_{FL} is the density of fluid, g is the gravity, T_{FL} is the temperature, κ is the thermal diffusivity, k_{FS} is the solutal diffusivity, κ_{FT} is the Dufour coefficient, C_{FL} is the concentration, α_{FT} is the thermal expansion coefficient α_{FS} is the solutal analog of α_{FT} , ϕ_{Fm} is the porosity of the porous medium, K is the permeability of the porous medium, μ_{Fm} is porous medium effective viscosity, $A = \frac{(\rho_{FLO} C_{FP})_m}{(\rho_{FL} C_{FP})_f}$ is the heat capacities ratio, C_{FP} is the specific heat. In the porous medium the quantities are referred by the subscripts p .

LINEAR STABILITY THEORY

Consider the basic state, in the composite layer system the fluid layer is stable and also assumed it is inactive, means quiescent. Therefore there is no motion, so, set the velocity vector $\vec{q}$ is zero, where, the mass and heat are transferred only by conduction. Let $\vec{V} = [u, v, w] = 0$ and $[T_{FL}, P_{FL}, C_{FL}, \rho_{FL}] = [T_{FLb}(z_{FL}), P_{FLb}(z_{FL}), C_{FLb}(z_{FL}), \rho_{FLb}(z_{FL})]$ for sparsely porous layer, $\vec{V}_{Fm} = [u_{Fm}, v_{Fm}, w_{Fm}] = 0$ and $[T_{Fm}, P_{Fm}, C_{Fm}, \rho_{Fm}] = [T_{Fmb}(z_{Fm}), P_{Fmb}(z_{Fm}), C_{Fmb}(z_{Fm}), \rho_{Fmb}(z_{Fm})]$ finally we get temperature and concentration distributions for both fluid and porous layer by substituting the above basic states to equations (1) to (10).

Temperature and concentration distribution in fluid layer ($0 \leq z \leq d$)

$$T_{FLb}(ZFL) = \frac{(T_{FLu} - T_{FLO})Z_{FL}}{d_{FL}} + T_0 \quad (11)$$

$$C_{FLb}(ZFL) = \frac{(C_{FLu} - C_{FLO})Z_{FL}}{d_{FL}} + C_0 \quad (12)$$

Temperature and concentration distribution in porous layer ($-d_p \leq z_p \leq 0$)

$$T_{Fmb}(ZFm) = \frac{(T_{Fm0} - T_{FmL})Z_{Fm}}{d_{Fm}} + T_0 \quad (13)$$

$$C_{Fmb}(ZFm) = \frac{(C_{Fm0} - C_{FmL})Z_{Fm}}{d_{Fm}} + C_0 \quad (14)$$

Where,

$$T_0 = \frac{k_{FL} d_{FL} T_{FLu} + d_{FL} K_{FLm} T_{FL}}{K_{FL} d_{FLm} + d_{FL} k_{FLm}} + \frac{d_{FLm} k_{FLT} C_{FLu} + k_{FL} T_{FLm} d_{FL} C_{FL}}{k d_p + d_{FL} k_p} - C_0 \left(\frac{d_{FL} K_{FL} T_{FLm} + d_{FLm} k_{FLT}}{k_{FL} d_{FLm} + d_{FL} k_{FLm}} \right)$$

And

$$C_0 = \frac{k_{FLs} C_{FLu} d_{FLm} + d_{FL} k_{FSm} C_{FmL}}{d_{FL} k_{FSm} + d_{Fm} k_{Fs}}$$

are the interface temperature and concentrations respectively. In order to analyze the stability of the basic state solution, an infinitesimal perturbations are introduced and the perturbed quantities are of the form.

For fluid layer

$$\left[\vec{V}_{FL}, P_{FL}, T_{FL}, C_{FL}, \rho_{FL} \right] = \left\{ \left[0, P_{FLb}(z_{FL}), T_{FLb}(z_{FL}), C_{FLb}(z_{FL}), \rho_{FLb}(z_{FL}) \right] + [\vec{q}, P', \theta, S, \rho'] \right\}$$

For porous layers

$$\left[\vec{V}_{Fm}, P_{Fm}, T_{Fm}, C_{Fm}, \rho_{Fm} \right] = \left\{ \left[0, P_{Fmb}(z_{Fm}), T_{Fmb}(z_{Fm}), C_{Fmb}(z_{Fm}), \rho_{Fmb} \right] + [\vec{q}_{Fm}, P'_{Fm}, \theta_{Fm}, S_{Fm}, \rho'_{Fm}] \right\}$$

Substitute equations (15) and (16) into equations (1) – (10) and apply two times curl on the momentum equations to eliminate the pressure term. The resulting equations are linearized for (1) – (10), then they are non-dimensionalized, the variables using $\frac{d^2}{k}$, $\frac{k}{d}$, $C_0 - C_{uf}$, d and $T_0 - T_{uf}$, for time, velocity, concentration, length and temperature respectively in region – 1 and $\frac{d_p^2}{k_p}$, $\frac{k_p}{d_p}$, $C_{Lf} - C_0$, d_p and $T_{Lf} - T_0$ are the corresponding scale factors in region – 2. Applying normal mode expansions to the perturbed non-dimensionalised quantities (following Sumithra [21]) result in the following ordinary differential equations,

For region – 1 (Fluid layer)

$$\left[D_{FL}^2 - a_{FL}^2 + \frac{n}{p_r} \right] (D_{FL}^2 - a_{FL}^2) W_{FL} = R_{FL} a_{FL}^2 \theta_{FL} - R_{FLs} a_{FL}^2 S_{FL} \quad (17)$$

$$(D_{FL}^2 - a_{FL}^2 + n) \theta_{FL} + W_{FL} + D_{FLr} (D_{FL}^2 - a_{FL}^2) S_{FL} = 0 \quad (18)$$

$$[\tau_{FL} (D_{FL}^2 - a_{FL}^2) + n] S_{FL} + W_{FL} = 0 \quad (19)$$

For region – 2 (porous layer)

$$\left[\hat{\mu}_{Fm} \beta_{Fm}^2 (D_{Fm}^2 - a_{Fm}^2) + \frac{n_p \beta^2}{p_{rp}} - 1 \right] (D_{Fm}^2 - a_{Fm}^2) W_{Fm} = R_{Fm} a_{Fm}^2 \theta_{Fm} - R_{FSm} a_{Fm}^2 S_{Fm} \quad (20)$$

$$(D_{Fm}^2 - a_{Fm}^2 + n_p A) \theta_{Fm} + W_{Fm} + D_{Fmr} (D_{Fm}^2 - a_{Fm}^2) S_{Fm} = 0 \quad (21)$$

$$[\tau_{Fm} (D_{Fm}^2 - a_{Fm}^2) + \phi n_p] S_{Fm} + W_{Fm} = 0 \quad (22)$$

For a particular set of various parameters, the principle of exchange instability is set up to be exactly true for fluid or porous layer problems separately and it is assumed that the principle holds for this case also, so, letting $n = n_p = 0$, equations (17) – (22) becomes

For region – 1 (Fluid layer)

$$(D_{FL}^2 - a_{FL}^2)^2 W_{FL} = R_{FL} a_{FL}^2 \theta_{FL} - R_{FLS} a_{FL}^2 S_{FL} \quad (17)$$

$$(D_{FL}^2 - a_{FL}^2) \theta_{FL} + W_{FL} + D_{FLr} (D_{FL}^2 - a_{FL}^2) S_{FL} = 0 \quad (18)$$

$$[\tau_{FL} (D_{FL}^2 - a_{FL}^2)] S_{FL} + W_{FL} = 0 \quad (19)$$

For region – 2 (porous layer)

$$[\hat{\mu}_{Fm} \beta_{Fm}^2 (D_{Fm}^2 - a_{Fm}^2) - 1] (D_{Fm}^2 - a_{Fm}^2) W_{Fm} = R_{Fm} a_{Fm}^2 \theta_{Fm} - R_{Fsm} a_{Fm}^2 S_{Fm} \quad (20)$$

$$(D_{Fm}^2 - a_{Fm}^2) \theta_{Fm} + W_{Fm} + D_{Fmr} (D_{Fm}^2 - a_{Fm}^2) S_{Fm} = 0 \quad (21)$$

$$[\tau_{Fm} (D_{Fm}^2 - a_{Fm}^2)] S_{Fm} + W_{Fm} = 0 \quad (22)$$

The technique of regular perturbation method is used to solve the above differential equations with corresponding suitable boundary conditions.

BOUNDARY CONDITIONS

At the upper boundary, $z_{FL} = 1$

$$W_{FL}(z_{FL}) = 0, \quad D_{FL}^2 W_{FL}(z_{FL}) + M \theta_{FL}(z_{FL}) + M_S S_{FL}(z_{FL}) = 0, \quad D_{FL} \theta_{FL}(z_{FL}) = 0, \\ D_{FL} S_{FL}(z_{FL}) = 0$$

At the interface, $z_{FL} = 0$ and $z_{Fm} = 1$

$$\hat{T}_{FL} W_{FL}(z) = W_{Fm}(z_{Fm}), \quad \hat{T}_{FL} \hat{d}_{FL} D_{FL} W_{FL}(z) = D_{Fm} W_{Fm}(z_{Fm}), \quad \theta_{Fm}(z_{Fm}) = \hat{T} \theta_{Fm}(z_{Fm}), \\ D_{Fm} \theta_{Fm}(z_{Fm}) = D_{Fm} \theta_{Fm}(z_{Fm}), \quad S_{FL}(z_{FL}) = \hat{S}_{Fm} S_{Fm}(z_{Fm}), \quad D_{FL} S_{FL}(z_{FL}) = \\ D_{Fm} S_{Fm}(z_{Fm}) \\ \hat{T}_{FL} \hat{d}_{FL}^3 \beta_{FL}^2 [D_{FL}^3 W_{FL}(z) - 3a_{FL}^2 D_{FL} W_{FL}(z)] \\ = -D_{Fm} W_{Fm}(z_{Fm}) + \mu_{Fm} \beta_{Fm}^2 [D_{Fm}^2 W_{Fm}(z_{Fm}) - 3a_{Fm}^2 W_{Fm}(z_{Fm})] \\ \hat{T}_{FL} \hat{d}_{FL}^2 [(D_{FL}^2 + a_{FL}^2) W_{FL}(z)] = \hat{\mu}_{Fm} (D_{Fm}^2 + a_{Fm}^2) W_{Fm}(z_{Fm})$$

At the lower boundary, $z_{Fm} = 0$

$$W_{Fm}(z_{Fm}) = 0, \quad D_{Fm} W_{Fm}(z_{Fm}) = 0, \quad D_{Fm} \theta_{Fm}(z_{Fm}) = 0, \quad D_{Fm} S_{Fm}(z_{Fm}) = 0$$

Where,

$$\hat{d} = \frac{d_{Fm}}{d}, \quad \hat{k} = \frac{\hat{d}}{\hat{T}} = \frac{k_{Fm}}{k}, \quad \hat{k}_s = \frac{k_{Fsm}}{k_{FS}} = \frac{\hat{d}}{\hat{S}}, \quad \hat{T} = \left[\frac{T_{FL} - T_0}{T_0 - T_{FU}} \right], \quad \hat{S} = \left[\frac{C_{FL} - C_0}{C_0 - C_{FU}} \right]$$

REGULAR PERTURBATION METHOD

For the consistent mass and heat fluxes, expanding the physical parameters in terms of horizontal wave number "a" and "a_p"

$$\begin{bmatrix} W_{FL} \\ \theta_{FL} \\ S_{FL} \end{bmatrix} = \sum_{j=0}^{\infty} a^{2j} {}_{FL}f_r \begin{bmatrix} W_{FLS} \\ \theta_{FLS} \\ S_{FLS} \end{bmatrix} \& \begin{bmatrix} W_{Fm} \\ \theta_{Fm} \\ S_{Fm} \end{bmatrix} = \sum_{j=0}^{\infty} a^{2j} {}_{Fmr} \begin{bmatrix} W_{Fmj} \\ \theta_{Fmj} \\ S_{Fmj} \end{bmatrix} \quad (23)$$

Equation(23)are substituted into the equations (17) – (22)and considering only the coefficients of a^2 which are like powers. The equations at zero order are solved with respect to the corresponding zero order boundary conditions and the obtained the zero order final solution as given below.

$$W_{Fm0}(z_{Fm}) = 0, W_{FL0}(z_{FL}) = 0, S_{FL0}(z_{FL}) = \hat{S}, S_{Fm0}(z_{Fm}) = 1, \theta_{FL0} = \hat{T}, \theta_{Fm0}(z_{Fm}) = 1$$

The equations at first order are given below.

For region – 1(Fluid layer)

$$D_{FL}^4 W_{FL1}(z) - R_{FL} \hat{T}_{FL} + R_{FLS} \hat{S}_{FL} = 0 \quad (30)$$

$$D_{FL}^2 \theta_{FL1} + W_{FL1}(z) + D_{FLr}(D_{FL}^2 S_{FL1} - \hat{S}_{FL}) - \hat{T}_{FL} = 0 \quad (31)$$

$$\tau_{FL} D_{FL}^2 S_{FL1} + W_{FL1} - \tau_{FL} \hat{S}_{FL} = 0 \quad (32)$$

For region –2(porous layer)

$$\mu_{Fm} \hat{\beta}_{Fm}^2 D_{Fm}^4 W_{Fm1}(z_{Fm}) - D_{Fm}^2 W_{Fm1}(z_{Fm}) - R_{Fm} + R_{Fsm} = 0 \quad (33)$$

$$D_{Fm}^2 \theta_{Fm1}(z_{Fm}) + W_{Fm1}(z_{Fm}) + D_{Fm} D_{Fm}^2 S_{Fm1} - D_{Fm} - 1 = 0 \quad (34)$$

$$\tau_{Fm} D_{Fm}^2 S_{Fm1} + W_{Fm1} - \tau_{Fm} = 0 \quad (35)$$

Boundary conditions corresponding to first order equations in a^2 are as mentioned below.

At the upper boundary, $z_{FL} = 1$

$$W_{FL1}(z_{FL1}) = 0, \quad D_{FL}^2 W_{FL1}(z) + M \theta_0(z) + M_S S_0(z) = 0, \quad D_{FL} \theta_{FL1}(z_{FL}) = 0,$$

$$D_{FL} S_{FL1}(z_{FL}) = 0$$

At the interface, $z_{Fm} = 0$ and $z_{Fm} = 1$

$$\hat{T}_{Fm} W_{Fm1}(z_{Fm}) = \hat{d}_{Fm}^2 W_{Fm1}(z_{Fm}),$$

$$\hat{T}_{Fm} D_{Fm} W_{Fm1}(z_{Fm}) = \hat{d}_{Fm} D_{Fm} W_{Fm1}(z_{Fm}), \quad \theta_{Fm1}(z_{Fm}) = \hat{d}_{Fm}^2 \hat{T}_{Fm} \theta_{Fm1}(z_{Fm}),$$

$$D_{Fm} \theta_{Fm1}(z_{Fm}) = \hat{d}_{Fm}^2 D_{Fm} \theta_{Fm1}(z_{Fm}), \quad S_{Fm1}(z_{Fm}) = \hat{S}_{Fm} \hat{d}_{Fm}^2 S_{Fm1}(z_{Fm}),$$

$$D S_{Fm1}(z_{Fm}) = \hat{d}_{Fm}^2 D_{Fm} S_{Fm1}(z_{Fm})$$

$$\hat{T}_{Fm} \hat{d}_{Fm} \beta_{Fm}^2 D_{Fm}^3 W_{Fm1}(z_{Fm}) = -D_{Fm} W_{Fm1}(z_{Fm}) + \hat{\mu}_{Fm} \beta_{Fm}^2 [D_{Fm}^3 W_{Fm1}(z_{Fm})]$$

At the lower boundary, $z_{Fm} = 0$

$$W_{Fm1}(z_{Fm}) = 0, D_{Fm} W_{Fm1}(z_{Fm}) = 0, D_{Fm} \theta_{Fm1}(z_{Fm}) = 0, D_{Fm} S_{Fm1}(z_{Fm}) = 0$$

The equations (30) and (33) solved using the relevant boundary conditions then we get velocity distributions as below,

$$W_{FL1}(z_{FL}) = \left[\frac{\hat{T}_{FL} R_{FL} - \hat{S}_{FL} R_{FLS}}{24} \right] (C_1 + C_2 z_{FL} + C_3 z_{FL}^2 + C_4 z_{FL}^3 + z_{FL}^4)$$

$$W_{FL1}(z_{FL}) = C_5 + C_6 z_{FL} + C_7 e^{p z_{FL}} + C_8 e^{-p z_{FL}} + \frac{(R_{Fsm} - R_{Fm})z_{Fm}^2}{2} + \frac{(R_{Fsm} - R_{Fm})}{p^2}$$

$$C_1 = \frac{24 \hat{d}^2}{\hat{T}(\hat{T}R - \hat{S}R_S)e^{-p}} \left[C_7(1 - pe^{-p} - e^{-p}) + C_8 e^{-p}(-1 + p + e^{-p}) + \frac{e^{-p}(R_{sp} - R_p)}{2} \right]$$

$$C_2 = \frac{24 \hat{d}}{\hat{T}(\hat{T}R - \hat{S}R_S)e^{-p}} [C_7(p - pe^{-p}) + C_8 pe^{-p}(1 - e^{-p}) + e^{-p}(R_{sp} - R_p)]$$

$$C_3 = \frac{12 \hat{\mu}}{\hat{T}(\hat{T}R - \hat{S}R_S)e^{-p}} [C_7 P^2 + C_8 P^2 e^{-2p} + e^{-p}(R_{sp} - R_p)]$$

$$C_4 = \frac{4}{\hat{T} \hat{d} \beta^2 (\hat{T}R - \hat{S}R_S)e^{-p}} [C_7 p(e^{-p} + (p - 1)) + C_8 pe^{-2p}(1 - p - e^p) + Q]$$

$$C_5 = -C_7 - C_8 - \frac{(R_{sp} - R_p)}{p^2}, \quad C_6 = -p C_7 + p C_8, \quad Q = e^{-p}(R_{sp} - R_p)$$

$$C_7 = \frac{\hat{T}(\hat{T}R - \hat{S}R_S)e^{-p}(b_2 - (M\hat{T} + Ms\hat{S} + 12)a_2) + (R_{sp} - R_p)(a_3b_2 - a_2b_3)}{a_2b_1 - a_1b_2}$$

$$C_8 = \frac{\hat{T}(\hat{T}R - \hat{S}R_S)e^{-p}((M\hat{T} + Ms\hat{S} + 12)a_1 - b_1) + (R_{sp} - R_p)(a_1b_3 - a_3b_1)}{a_2b_1 - a_1b_2}$$

COMPATIBILITY CONDITION

The differential equations corresponding to concentration and temperature, along with corresponding boundary conditions gives the compatibility condition as below.

$$\left\{ \begin{aligned} & \int_0^1 W_{FL1}(z_{FL}) dz_{FL} - \frac{D_{FLr}}{\tau_{FL}} \int_0^1 W_{FL1}(z_{FL}) f(z_{FL}) dz_{FL} \\ & + \hat{d}_{Fm}^2 \int_0^1 W_{Fm1}(z_{Fm}) dz_{Fm} - \hat{d}_{Fm}^2 \frac{D_{Fmr}}{\tau_{Fm}} \int_0^1 W_{Fm1}(z_{Fm}) g(z_{Fm}) dz_{Fm} \end{aligned} \right\} = \hat{d}^2 + \hat{T} \quad (38)$$

Here $f(z_{FL})$ and $f(z_{Fm})$ take different forms according to the uniform and non-uniform temperature gradients.

Linear Temperature Profile (LTP)

$$f(z_{FL}) = 1 \text{ and } f(z_{Fm}) = 1$$

Parabolic Temperature Profile (PTP)

$$f(z_{FL}) = 2z_{FL} \text{ and } f(z_{Fm}) = 2z_{Fm}$$

Now to find Critical Rayleigh number substitute (36) and (37) in equation (38) then solving for the critical Rayleigh number, we get

$$R_L = \frac{\left[\frac{T_{FL} - T_0}{T_0 - T_{FU}} \right] + \hat{d}^2 + R_S \left[\frac{C_{FL} - C_0}{C_0 - C_{FU}} \right] (\Psi_1 - \lambda_1 \Psi_2)}{\left[\frac{T_{FL} - T_0}{T_0 - T_{FU}} \right] (\Psi_1 - \lambda_1 \Psi_2)}$$

$$R_p = \frac{\left[\frac{T_{Fm}-T_0}{T_0-T_{FU}} \right] + \hat{d}^2 + R_S \left[\frac{C_{Fm}-C_0}{C_0-C_{FU}} \right] (\Psi_1 - \lambda_1 \Psi_2)}{\left[\frac{T_{Fm}-T_0}{T_0-T_{FU}} \right] (\Psi_1 - \lambda_1 \Psi_2)}$$

Where,

$$H_1 = \frac{\beta^2 \hat{d}^3}{\hat{k}}, \Psi_1 = \frac{\tau - D_r}{120} + \frac{H_3}{H_2} + \frac{H_4}{H_2}, \Psi_2 = \frac{H_5}{H_2} + \frac{H_6}{H_2} + A_3, \mathbb{N} = (M \hat{T} + M s \hat{S} + 12)$$

$$H_2 = c_2 b_1 - c_1 b_2, \quad H_3 = A_1 \hat{T} * e^{-p} (b_2 - \mathbb{N} c_2), \quad H_4 = A_2 \hat{T} e^{-p} (\mathbb{N} c_1 - b_1)$$

$$H_5 = A_1 (c_3 b_2 - c_2 b_3), \quad H_6 = A_2 (c_1 b_3 - c_3 b_1), \quad H_7 = \frac{\tau - D_F}{24 \tau \hat{T} \hat{d} \beta^2}$$

$$A_1 = H_7 H_8 + H_9 H_{10} + H_{11} H_{12} + H_{11} H_{13} - H_{14} - H_{15} + H_{16} e^{-p} - 1)$$

$$A_2 = H_7 H_{17} + H_9 H_{18} + H_{11} H_{19} + H_{11} H_{20} - H_{14} + H_{15} + H_{16} e^{-p} - H_{16}$$

$$A_3 = H_7 + H_9 + \frac{H_{11}}{2} + \frac{H_{11} \hat{d}}{2} + \frac{1}{6}, \quad H_8 = \frac{(p-1+e^{-p})p}{e^{-p}}, \quad H_9 = \frac{(\tau - D_F) \hat{\mu}}{6 \tau \hat{T}}$$

$$H_{10} = \frac{p^2}{e^{-p}}, \quad H_{11} = \frac{(\tau - D_F) \hat{d}}{\tau \hat{T}}, \quad H_{12} = \frac{p(1-e^{-p})}{2e^{-p}}, \quad H_{13} = \frac{(1-pe^{-p}-e^{-p})\hat{d}}{e^{-p}}$$

$$H_{14} = \frac{\tau_m \hat{d}^2 - D_{FM} \hat{d}^2}{\tau_m}, \quad H_{15} = \frac{p(\tau_m \hat{d}^2 - D_{FM} \hat{d}^2)}{2\tau_m}, \quad H_{16} = \frac{(\tau_m \hat{d}^2 - D_{FM} \hat{d}^2)}{p\tau_m}$$

$$H_{17} = p(1-p-e^p)e^{-p}, \quad H_{18} = p^2 e^{-p}, \quad H_{19} = \frac{p(1-e^{-p})}{2}, \quad H_{20} = \hat{d}(p+e^{-p}) - \hat{d}$$

$$c_1 = 24 \hat{d}^2 (1 - e^{-p}(p-1)) + 24 \hat{d} p(1 - e^{-p}) + 12 \hat{\mu} p^2 + \frac{p^4 (p+P_1)}{\hat{d} \beta^2}, \quad P_1 = e^{-p} - 1$$

$$c_2 = 24 \hat{d}^2 (P_1 + p) e^{-p} + 24 p e^{-p} (1 - e^{-p}) + 12 \hat{\mu} p^2 e^{-2p} + \frac{4 p e^{-2p} (1 - p - e^p)}{d \beta^2}$$

$$c_3 = 12 \hat{d}^2 e^{-p} + 24 \hat{d} e^{-p} + 12 \hat{\mu} e^{-p} - \frac{4 e^{-p}}{\hat{d} \beta^2}, \quad b_3 = 24 \hat{\mu} e^{-p} - \frac{24 e^{-p}}{\hat{d} \beta^2}$$

$$b_1 = 24 \hat{\mu} p^2 + \frac{24 e^{-p}}{\hat{d} \beta^2}, \quad p = (\hat{\mu} \beta^2)^{-1/2}, \quad \beta = \sqrt{D a}, \quad b_2 = 24 \hat{\mu} p^2 e^{-2p} + \frac{24 p e^{-p}}{\hat{d} \beta^2}$$

RESULTS AND DISCUSSIONS

The influence of Non-Darcian effect and Diffusion – Thermo (Dufour effect) effect on Two-Component Rayleigh Benard Marangoni (TCRBM) convection in a composite layer, has been investigated analytically with regular perturbation techniques. The solutions are obtained in terms of most relevant various physical quantities such as the Darcy number, the thermal diffusivity ratio, ratio of solutal diffusivities, viscosity ratio, solutal Rayleigh number, Marangoni number, thermal Marangoni number and the Dufour parameter of both fluid and porous layer. Plot the graph Rayleigh number (R) versus depth ratio ($\hat{d}$) and for fixed values of other parameters such as $D_a = 0.0001, \hat{k} = 2, R_s = 1000, \hat{\mu} = 2, \tau = 0.25, \tau_m = 0.75, D_F = 1, D_{FM} = 1, \hat{S} = 1, \hat{T} = 1, M = 10$ and $M_s = 1000$ are presented.

Figure – 1, illustrates that the influence of Darcy number for $D_a = 0.0001, 0.0019$ and 0.01 on TCRBM convection. The Rayleigh number de-escalates as escalating the values of $D_a = [K/d_m^2]$, the composite system is destabilized by the Darcy number.

Figure – 2, illustrates the influence of Diffusivity ratio $\hat{k} = 0.5, 1$ and 2 on TCRBM convection in a composite layer. For fixed value of depth ratio $\hat{d}$, Rayleigh number R rises up it means flow resistance increases for increased values of $\hat{k}$, hence the effect of this parameter is to stabilize the system and to postpone the onset of TCRBM convection. And the curves are converging at one end and diverges at other end.

Figure – 3 illustrates the influence of ratio of viscosity $\hat{\mu} = 1, 1.5$ and 2 on TCRBM convection. For fixed value of depth ratio $\hat{d}$, Rayleigh number R falls down for increased values of $\hat{\mu}$, hence the effect of this parameter is to destabilize the system and to prepone the onset of TCRBM convection. And also the curves are converging at one end and diverges at another end. Figure – 4 illustrates the influence of Solutal Rayleigh number $R_s = 250, 250.1$ and 250.2 on TCRB convection, Rayleigh number R rises up, it means flow resistance increases for increased values of R_s . Hence TCRBM convection can be postponed with the large values of Solutal Rayleigh number. And the curves are diverges at the both ends.

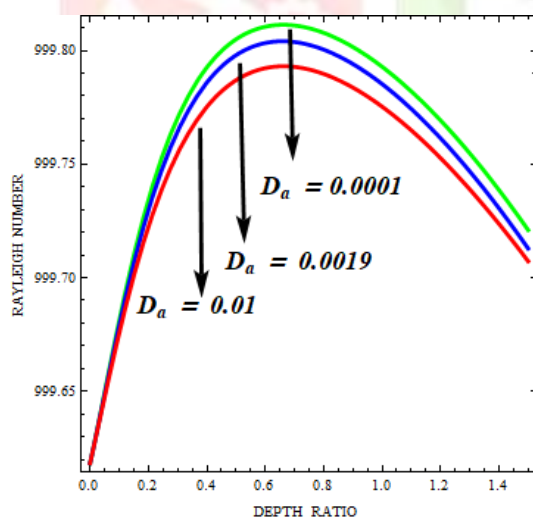


Figure – 1: The impact of D_a on Rayleigh Number (R)

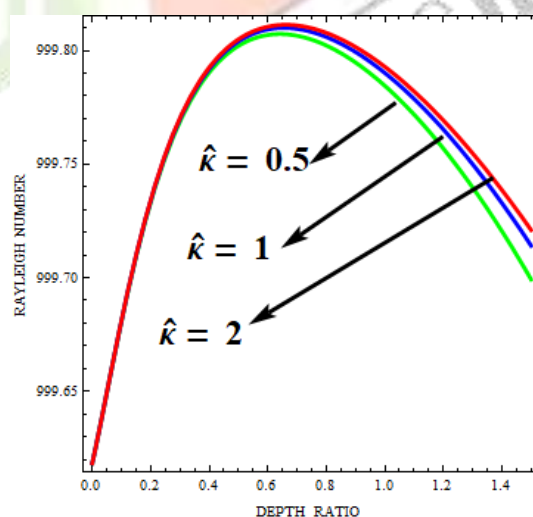


Figure – 2: The impact of $\hat{k}$ on Rayleigh Number (R)

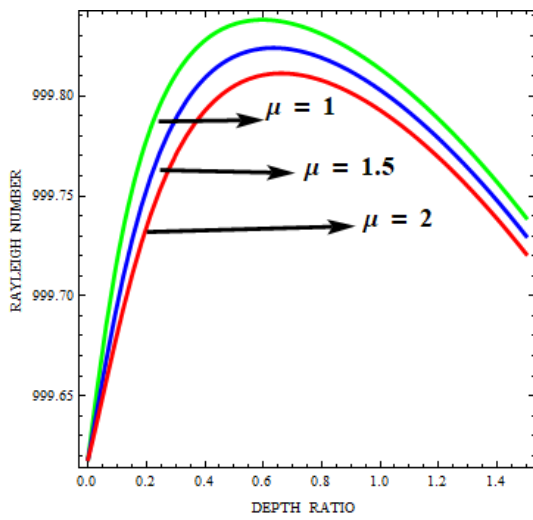


Figure – 3: The impact of μ on Rayleigh Number(R)

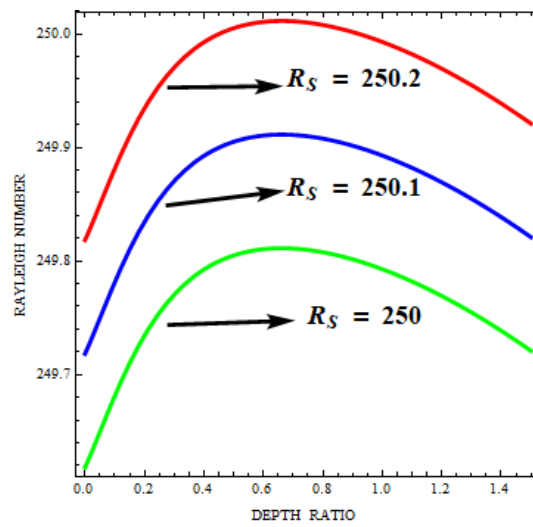


Figure – 4: The impact of R_s on Rayleigh Number(R)

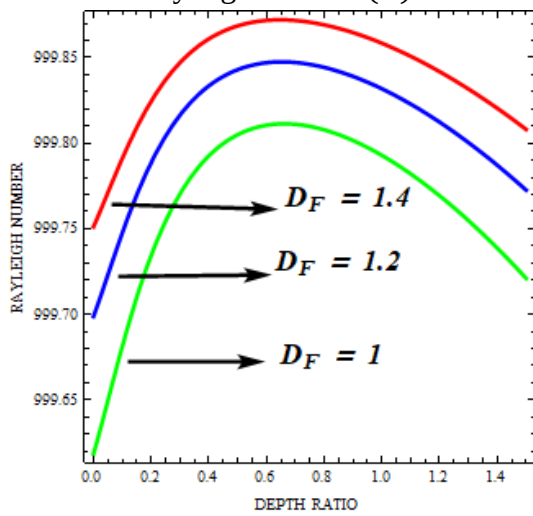


Figure – 5: The impact of D_F on Rayleigh Number(R)

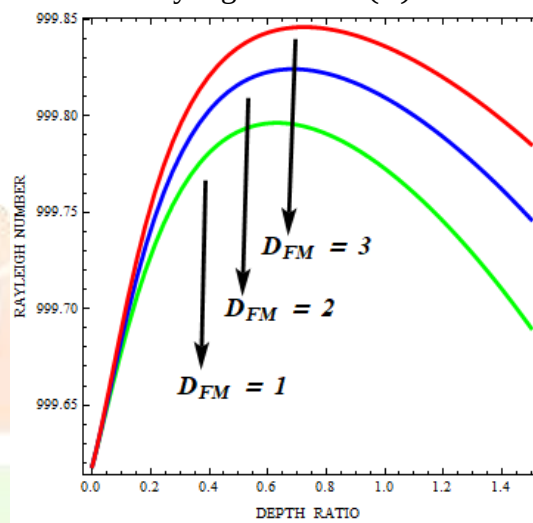


Figure – 6: The impact of D_{FM} on Rayleigh Number(R)

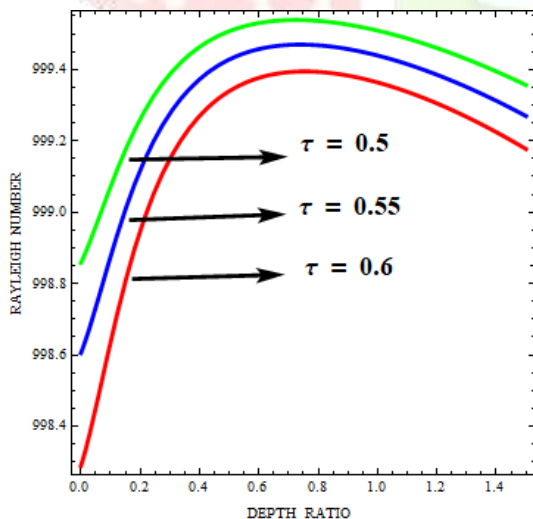


Figure – 7: The impact of τ on Rayleigh Number(R)

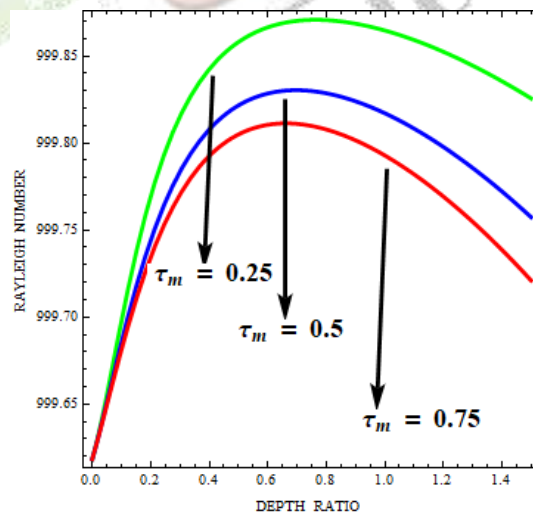


Figure – 8: The impact of τ_m on Rayleigh Number(R)

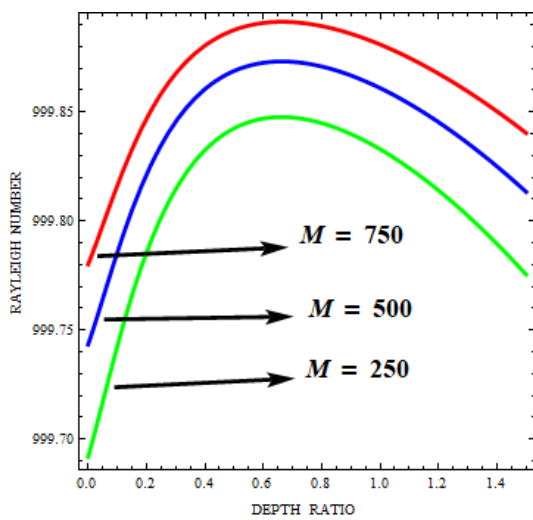


Figure – 9: The impact of M on Rayleigh Number(R)

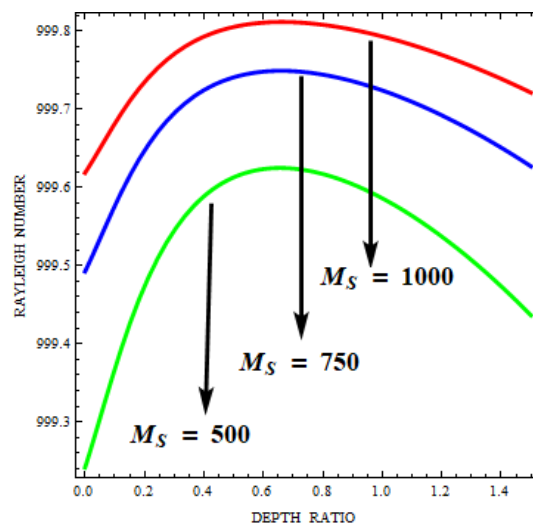


Figure – 10: The impact of M_s on Rayleigh Number(R)

Figure – 5 illustrates the influence of Dufour parameter of the fluid layer (Region-1) $D_F = 1, 1.2$ and 1.4 on TCRBM convection in composite layer. Here the critical Rayleigh number R rises up it means flow resistance increases for increased values of D_F . That is the impact of the Dufour coefficient is to stabilize the two component system and to prepone the onset of TCRBM convection. Figure – 6 demonstrates the significance of Dufour parameter of the porous layer (Region – 2) $D_{FM} = 1, 2$ and 3 plays same role on the onset of TCRBM Convection. The higher values of this parameter prepones the onset of TCRBM Convection and hence its effect is to stabilize the system.

Figure – 7 illustrates the influence of the ratio of salinity diffusivity to thermal diffusivity of the fluid layer $\tau = 0.5, 0.55$ and 0.6 on TCRBM convection in composite layer. Here the critical Rayleigh number R falls down it means flow resistance decreases for increased values of τ . And also for porous layer $\tau_m = 0.25, 0.5$ and 0.75 on TCRBM convection in composite layer plays a same role on the onset of TCRBM Convection is portrayed in Figure – 8. The higher values of this parameter postpones the onset of TCRBM Convection and hence its effect is to destabilize the system.

Figure – 9 illustrates the influence of thermal Marangoni number $M = 250, 500$ and 750 on TCRBM convection in composite layer. Here the critical Rayleigh number R rises up it means flow resistance increases for increased values of M . That is the impact of the Marangoni number is to stabilize the two component system and to prepone the onset of TCRBM convection. Figure – 10 demonstrates the significance of Solutal Marangoni number $M_s = 500, 750$ and 1000 plays same role on the onset of TCRBM Convection. The higher values of this parameter prepones the onset of TCRBM Convection and hence its effect is to stabilize the system.

CONCLUSION

TCRBM Convection in a Composite layered system with Dufour effect is solved in closed form using Regular Perturbation method and the following deductions are made from the study.

- All the parameters except for the Solutal Rayleigh number, Marangoni number and Thermal Marangoni number are effective for smaller values of the depth ratio, that is, they are all effective for fluid layer dominant composite layers
- The impact of viscosity ratio(μ), Darcy number(D_a) and the ratio of salinity diffusivity to thermal diffusivity of the fluid layer and porous layer (τ, τ_m) is to prepone the onset of TCRBM convection and larger values of these parameters are preferred in the situations demanding more heat transfer.
- The effect of Dufour parameters for both fluid and porous layer (D_F, D_{FM}), thermal diffusivity ratio($\hat{k}$) and Solutal Rayleigh number(R_s), thermal Marangoni number (M) and solutal Marangoni number (M_s) is to postpone the onset of TCRBM Convection and larger values of these parameter is preferred in the situations where in the convection has to be controlled.
- Dufour effect (Diffusion-Thermo) play a vital role on TCRBM convection. The effect of Dufour parameter in both fluid and porous layer is stabilizing the composite system.

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